of operation. This is done so that there will be minimal radiation loss via such a parallel plate mode in the region of coupling between closed rectangular waveguide and the open groove guide structure. The second step consists of increasing this dimension from the reduced value to the final value for the groove guide design. It should be noted that the plate separation in the groove region is equal to the wide dimension of rectangular waveguide. This was done solely as a matter of convenience.

Standard laboratory measurements of transducer insertion loss and input VSWR have been made, utilizing a sliding short in groove guide and substitution techniques, and the results are shown in Fig. 2. The physical length of the transducer, which was not optimized, accounts for approximately 0.2 dB of the insertion loss. It is felt, therefore, that a more efficient coupling design would reduce the coupling length (now approximately 20 wavelengths) by a factor of two and thereby reduce the insertion loss to a value below 0.2 dB over the entire frequency range. Thus, the transducer insertion loss would be equivalent to less than six inches of rectangular waveguide operating in the same frequency range.

It has been shown [1]–[5] that the dispersion relation for groove guide has the familiar form

$$\lambda_g = \frac{\lambda}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}}$$

where

$$\lambda_c = \frac{2\pi}{k_c};$$

$$k_c = \sqrt{\left(\frac{\pi}{b}\right)^2 + k_{yA}^2}$$

$$= \sqrt{\left(\frac{\pi}{b'}\right)^2 - |k_{yB}|^2}$$

where k_{yA} is the y-directed wavenumber in the groove region and $|k_{yB}|$ is the y-directed wavenumber in the outer region giving the decay rate in nepers per unit length (in y). Now k_{yA} and/or $|k_{yB}|$ are determined by the transverse procedure for a structure with specific dimensions. An experimental confirmation of the approximate theory was obtained for the case $a/b=3$ using a groove guide tunable reaction type cavity having $b=0.2800$ inch and $b'=0.270$ inch and a plate width of approximately 5 inches.

The cavity was excited by a capacitive iris in a transverse shorting plate at the input end and terminated in a shorting plate whose longitudinal position could be adjusted. The guide wavelength at any frequency was determined by measuring the distance between successive resonant positions of the tunable short. In Table I various values of guide wavelength are compared at a number of frequencies. The first two columns of guide wavelength permit a comparison of the measured values with those theoretically calculated. In the last column are predicted values of λ_g for a parallel plate mode if it were to exist rather than the bound groove guide mode. It is easily seen that the measured values of guide wavelength correspond to the predicted values for groove guide and at the same time differ greatly from the values predicted for parallel plate guide.

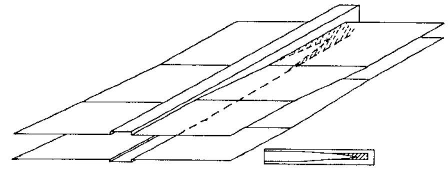

Fig. 1. Rectangular to groove guide transducer.

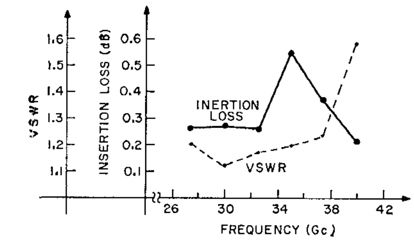

Fig. 2. Transducer performance.

TABLE I

f(Gc/s)	Measured λ_g(inches) G.G.	Calculated λ_g(inches) G.G.	Calculated λ_g(inches) P.P.
27.62	0 6803	0.6763	0.7055
28.82	0 6156	0.6117	0 6330
30.02	0.5622	0.5608	0.5771
31.96	0 5002	0.4974	0.5086
33.90	0 4492	0.4490	0.4572
35.86	0 4092	0.4102	0.4165
38.00	0.3774	0 3759	0.3807

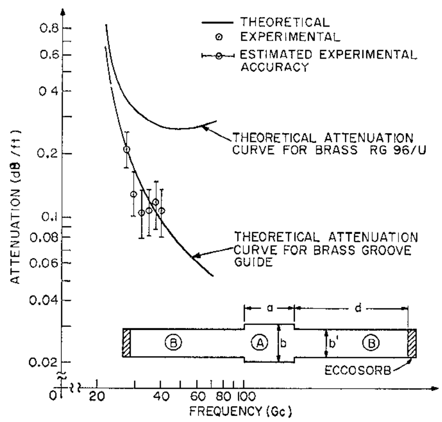

Fig. 3. Attenuation vs. frequency; groove guide no. 2 (machined brass); (a =0.140 inch, b =0.280 inch, b'=0.250 inch, d =5.0 inch).

The attenuation constant was measured using the groove guide sliding short and substitution. The groove guide under measurement was fabricated from machined brass and is shown in Fig. 3 with the experimental results.

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REFERENCES

[1] J. M. Ruddy, "The groove guide," Polytechnic Inst. of Brooklyn, N. Y., Progress Rept. 26 to Joint Services TAC, Rept. R-452.26-64, pp. 79–82, September 1964.
[2] ——, "Preliminary analysis of low loss groove guide," Polytechnic Inst. of Brooklyn, N. Y., Memo 78, Rept. PIBMRI-1127-63, February 1963.
[3] ——, "Grove guide," Polytechnic Inst. of Brooklyn, N. Y., Memo 89, Repts. PIBMRI-1177-63, July 1963.
[4] ——, "The groove guide," Polytechnic Inst. of Brooklyn, N. Y., Rept. PIBMRI-1240-64, October 1964.
[5] J. W. E. Griemsmann, "Groove guide," *1964 Proc. Symp. on Quasi-Optics*, vol. XIV, p. 365.
[6] L. Birenbaum and J. W. E. Griemsmann. "A low loss H-guide for millimeter wavelengths," *1959 Proc Symp. on Millimeter Waves*, vol. IX, p. 543.

Field Equations in Cylindrical Coordinates for Gyroelectric Media with Sources

For a gyroelectric medium characterized by a tensorial relative permittivity of the form

$$\overline{\overline{K}} = \begin{bmatrix} K_{11} & jK_{12} & 0 \\ -jK_{12} & K_{11} & 0 \\ 0 & 0 & K_{33} \end{bmatrix}$$

Maxwell's equations may be written as

$$\nabla \times \overline{H} = J_{ze}\bar{u}_z + \overline{J}_{te} + j\omega\epsilon_0\overline{\overline{K}}\cdot\overline{E}$$

$$\nabla \times \overline{E} = J_{zm}\bar{u}_z + \overline{J}_{tm} - j\omega\mu_0\overline{H}$$

where both electric and magnetic currents, designated by subscripts e and m, respectively, have been included. The currents have been separated into longitudinal and transverse components, designated by subscript z and t, respectively, with $\bar{u}_z$ representing a unit vector in the longitudinal direction. Throughout it is assumed that the fields and the currents have a time and z variation of the form

$$\exp(j\omega t - \gamma z).$$

Through the use of vector and tensor algebra, these two equations can be reduced to a set of wave equations which are in general coupled [1]. In matrix form they can be written as

$$\nabla_t^2 F + AF = J. \tag{1}$$

F and J are both two-dimensional vectors

$$F = \begin{bmatrix} E_z \\ H_z \end{bmatrix} \quad J = \begin{bmatrix} J_1 \\ J_2 \end{bmatrix}.$$

A is a 2×2 matrix having elements a_{11}, a_{12}, a_{21}, and a_{22}; and ∇_t is the transverse part of the del operator

$$\nabla_t = \nabla - \frac{\partial}{\partial z}\bar{u}_z.$$

Defining

$$k_0^2 = \omega^2\mu_0\epsilon_0$$

$$k_1^2 = -k_0^2K_{12}$$

and

$$k^2 = \gamma^2 + k_0^2K_{11}$$

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then

$$a_{11} = k^2K_{33}/K_{11}$$
$$a_{12} = -\omega\mu_0\gamma K_{12}/K_{11}$$
$$a_{21} = \omega\epsilon_0\gamma K_{12}K_{33}/K_{11}$$
$$a_{22} = k^2 + k_1^2K_{12}/K_{11}.$$

The currents J_1 and J_2 are divided into two parts, one part arising from the longitudinal currents and the other part from the transverse currents

$$J_1 = J_{1z} + J_{1t}$$

where

$$J_{1z} = \frac{j}{\omega}\left[\frac{k^2}{\epsilon_0K_{11}}J_{ze} - \frac{a_{12}}{\mu_0}J_{zm}\right]$$

and

$$J_{1t} = \nabla_t\cdot\left[\bar{u}_z \times \bar{J}_{tm} - j\frac{\gamma}{\omega\epsilon_0K_{11}}\bar{J}_{te}\right].$$

Similarly

$$J_2 = J_{2z} + J_{2t}$$

where

$$J_{2z} = -\frac{j}{\omega\mu_0}\left[a_{22}J_{zm} + a_{12}J_{ze}\right]$$

and

$$J_{2t} = \nabla_t\cdot\left[\bar{u}_z \times \bar{J}_{te} - j\frac{K_{12}}{K_{11}}\bar{J}_{te} + j\frac{\gamma}{\omega\mu_0}\bar{J}_{tm}\right].$$

The transverse fields can be expressed in terms of the longitudinal fields and transverse currents as follows:

$$\begin{aligned}\bar{E}_t = \frac{1}{(k^4 - k_1^4)}\{&\nabla_t(-\gamma k^2E_z - \omega\mu_0k_1^2H_z) \\ &+ j\bar{u}_z \times \nabla_t(\gamma k_1^2E_z + \omega\mu_0k^2H_z) \\ &+ \bar{u}_z \times (\gamma k^2\bar{J}_{tm} + \omega\mu_0k_1^2\bar{J}_{te}) \\ &+ j(\omega\mu_0k^2\bar{J}_{te} + \gamma k_1^2\bar{J}_{tm})\}\end{aligned} \quad (2a)$$

$$\begin{aligned}\bar{H}_t = \frac{1}{(k^4 - k_1^4)}\{&\nabla_t(-\gamma k^2H_z + \gamma^2\omega\epsilon_0K_{12}E_z) \\ &+ j\bar{u}_z \times \nabla_t[\gamma k_1^2H_z - \omega\epsilon_0(k_1^2K_{12} \\ &+ k^2K_{11})E_z] \\ &+ \bar{u}_z \times (\gamma k^2\bar{J}_{te} - \omega\epsilon_0\gamma^2K_{12}\bar{J}_{tm}) \\ &+ j[k_1^2\gamma\bar{J}_{te} - \omega\epsilon_0(k_1^2K_{12} \\ &+ k^2K_{11})\bar{J}_{tm}]\}.\end{aligned} \quad (2b)$$

If $a_{12} = a_{21} = 0$ then (1) represents two uncoupled equations which can be solved for E_z and H_z; these expressions for E_z and H_z can be used in (2a) and (2b) to determine the transverse fields. The matrix elements a_{12} and a_{21} are both zero if $K_{12} = 0$ or $\gamma = 0$.

If a_{12} and a_{21} are not both zero, then (1) can be uncoupled by a suitable linear transformation of the form

$$F = TU \quad (3)$$

where T is a 2×2 matrix and U is a two-dimensional vector.

Substituting (3) into (1) and premultiplying by T^{-1} gives

$$\nabla_t^2T^{-1}TU + T^{-1}ATU = T^{-1}J.$$

Using the theory of similarity transformations, it is possible to find a matrix T which will diagonalize the matrix A, giving

$$T^{-1}AT = D(p_1^2, p_2^2)$$

where p_1^2 and p_2^2 are two solutions of the characteristic equation of A

$$\det(A - p^2I) = 0. \quad (4)$$

Making such a transformation (1) becomes

$$\nabla_t^2U + D(p_1^2, p_2^2)U = T^{-1}J. \quad (5)$$

Solutions of (4) are available in the literature in terms of the elements of $\overline{\overline{K}}$ [2].

If neither a_{12} nor a_{21} are zero, and if $p_1^2 \neq p_2^2$, then an appropriate matrix T, giving the same transformation used by Kales [3] in a similar derivation for a source free ferrite, is

$$T = \begin{bmatrix} \frac{(p_1^2 - a_{22})p_1^2}{a_{21}} & \frac{(p_2^2 - a_{22})p_2^2}{a_{21}} \\ p_1^2 & p_2^2 \end{bmatrix}$$

the inverse of which is

$$T^{-1} = \begin{bmatrix} \frac{a_{21}}{p_1^2(p_1^2 - p_2^2)} & \frac{(p_2^2 - a_{22})}{p_1^2(p_2^2 - p_1^2)} \\ \frac{a_{21}}{p_2^2(p_2^2 - p_1^2)} & \frac{p_1^2 - a_{22}}{p_2^2(p_1^2 - p_2^2)} \end{bmatrix}.$$

The longitudinal fields are now obtainable from the solutions of (5) through the use of (3). The transverse field expressions written in terms of the solutions of (5) and the transverse currents are

$$\begin{aligned}\bar{E}_t = &\frac{1}{\omega\epsilon_0K_{12}}\nabla_t[(k^2 - p_1^2)U_1 + (k^2 - p_2^2)U_2] \\ &+ j\omega\mu_0\bar{u}_z \times \nabla_t(U_1 + U_2) \\ &+ \frac{1}{k^4 - k_1^4}\{\bar{u}_z \times (\gamma k^2\bar{J}_{tm} + \omega\mu_0k_1^2\bar{J}_{te}) \\ &+ j(\omega\mu_0k^2\bar{J}_{te} + \gamma k_1^2\bar{J}_{tm})\}\end{aligned}$$

$$\begin{aligned}\bar{H}_t = &-\gamma\nabla_t(U_1 + U_2) - j\frac{K_{11}}{\gamma K_{12}}\bar{u}_z \\ &\times \nabla_t[(p_1^2 - a_{22})U_1 + (p_2^2 - a_{22})U_2] \\ &+ \frac{1}{k^4 - k_1^4}\{\bar{u}_z \times (\gamma k^2\bar{J}_{te} - \omega\epsilon_0\gamma^2K_{12}\bar{J}_{tm}) \\ &+ j[k_1^2\gamma\bar{J}_{te} - \omega\epsilon_0(k_1^2K_{12} + k^2K_{11})\bar{J}_{tm}]\}.\end{aligned}$$

Making a transformation from a gyroelectric medium to a gyromagnetic medium the equations presented here reduce to those obtained by Rosenbaum and Coleman [4] for a ferrite containing longitudinal electric currents only.

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REFERENCES

[1] R. K. Likuski, "Free and driven modes in anisotropic plasma guides and resonators," Elec. Engrg. Research Lab., University of Illinois, Urbana, Tech. Rept. AL-TOR-64-157, July 1964.
[2] W. P. Allis, S. J. Buchsbaum, and A. Bers, *Waves in Anisotropic Plasmas*. Cambridge, Mass.: M.I.T. Press, 1963, pp. 155–156.
[3] M. L. Kales, "Modes in wave guides containing ferrites," *J. Appl. Phys.*, vol. 24, pp. 604–608, May 1963.
[4] F. J. Rosenbaum and P. D. Coleman, "Cerenkov radiation in anisotropic ferrites," *IEEE Trans. on Microwave Theory and Techniques*, vol. MTT-11, pp. 302–311, October 1963.

On the Superheterodyne Method of Microwave Noise Measurements

One of the frequently used methods in microwave oscillator noise measurements is the superheterodyne method. This correspondence is not intended to describe this method in detail, but to call attention to the possibility of pointing out one measurement error. Every one who is interested in microwave oscillator measurements can find fundamental information given in [1].

The superheterodyne method is assumed to be a substitution method, and therefore it is not necessary to know the mixer crystal noise [1], [2]. It will be shown that some conditions have to be fulfilled if the superheterodyne method is to become a substitution method which gives correct values of the oscillator noise power.

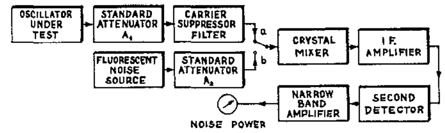

Fig. 1. Schematic diagram of the superheterodyne method noise measuring circuit.

The schematic diagram of the noise measuring equipment is given in Fig. 1. When the oscillator under test is connected to the input of the superheterodyne receiver [case (a)], it produces a reading on the output meter

$$N_1 = \left(\frac{N_o}{L_cA_1} + N_{c1} + N_a\right)G_a \quad (1)^{1,2}$$

where

N_o = noise power of the oscillator under test
N_{c1} = noise power contributed by the mixer crystals for the case (a)
N_a = noise power contributed by the IF amplifier
G_a = amplifier gain
L_c = conversion loss of mixer crystals
A_1 = attenuation factor of the standard attenuator in the oscillator arm.

When the tested oscillator is removed and a known amount of noise from the noise source is added [case (b)] the reading is

$$N_2 = \left(\frac{N_n}{L_cA_2} + N_{c2} + N_a\right)G_a \quad (2)$$

where

N_n = output noise power of the noise source
N_{c2} = noise power contributed by the mixer crystals for case (b)
A_2 = attenuation factor of the standard attenuator in the noise source arm.

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[1] The exact form of (1) and others similar in this correspondence is

$$N = \left(\frac{N_o - kT_o}{L_cA_1} + N_c + N_a\right)G_a.$$

However, usually $N_o \gg kT_0$.

[2] All noise powers N is this correspondence are considered as noise powers in unity bandwidth.